CS 4814: Homework 7

due Friday 10/30, 11:59pm
(late submission until **Monday 11/2 11:59pm**)

Each question is worth 20 points.

Question 1

Consider the following decision problem:

> MIN-CIRCUIT: Given an n-input 1-output Boolean circuit C, decide if C is a circuit of minimum size computing the function $x \mapsto C(x)$.

Show that MIN-CIRCUIT $\in \mathrm{coNP}^{\mathrm{SAT}}$. (In fact, this problem is $\mathrm{coNP}^{\mathrm{SAT}}$-complete with respect to polynomial-time Karp reductions.)

Question 2

We say that an n-input 1-output circuit C is a *formula* if every gate in C has out-degree 1.

a. Show that every function $f\colon \{0,1\}^n$ has a formula of size $O(2^n)$.

b. Show that for every large enough $n \in \mathbb{N}$, there exists a function $f\colon \{0,1\}^n \to \{0,1\}$ that requires formulas of size $\Omega(2^n / \log n)$.

Question 3

Show that every function $f\colon \{0,1\}^n \to \{0,1\}$ has a circuit of size $O(2^n/n)$. (Also see the hints for exercise 6.1 in the textbook.)