

# CS 4814: Homework 6

due Friday 10/23, 11:59pm  
(late submission until **Monday 10/26 11:59pm**)

Each question is worth 20 points.

## Question 1

Show that if  $P = NP$  then one-way functions do not exist.

## Question 2

Show that for every  $f: \{0, 1\}^n \rightarrow \{0, 1\}$  and  $S \in \mathbb{N}$ , the function  $f$  has a size- $S$  Boolean circuit if and only if  $f$  is computed by a length- $S$  Boolean straight-line program. (You can find the definition of Boolean straight-line programs in section 6.1 of the textbook.)

## Question 3

The function  $\text{MAJORITY}_n: \{0, 1\}^n \rightarrow \{0, 1\}$  returns 1 if *at least*  $n/2$  of its inputs are 1, and 0 if *fewer than*  $n/2$  of them are.

Show that  $\text{MAJORITY}_n$  has a circuits of size  $O(n^2)$ .