CS 4814: Homework 4

due 09/23, 11:59pm

This homework is about the relationship of P and NP in general and NP intermediate problems in particular. Recall that SAT is the problem of deciding the satisfiability of boolean formulas (say formulas in conjunctive normal form). This problem is NP-complete and has $2^{O(n)}$-time algorithms (enumerate all assignments to the variables). It is widely believed that SAT does not have $2^{o(n)}$-time (almost as strongly as P $\neq$ NP is believed). This conjecture is related to the exponential-time hypothesis.[1]

Also recall from the lecture the problem $\mathrm{SAT}_H = \{x01^{|x|^{H(|x|)}} \mid x \in \mathrm{SAT}\}$, where $H\colon \mathbb{N} \to \mathbb{N}$.

Each question is worth 30 points.

Question 1

Let $H(n) = \lfloor \log \log n \rfloor$.

Show each of the following statements:

a. If SAT does not have $2^{o(n)}$-time algorithms, then NP-complete problems do not have $2^{n^{o(1)}}$-time algorithms.

b. The problem SAT_H has a $2^{n^{o(1)}}$-time algorithm.

c. If SAT does not have $2^{o(n)}$-time algorithms, the problem SAT_H has intermediate complexity, i.e., it is neither in P nor NP-complete.

Question 2

a. Let $f\colon \{0,1\}^* \to \{0,1\}^*$ be a polynomial-time Karp reduction from SAT to itself (so that every string $x \in \{0,1\}^*$ satisfies $x \in \mathrm{SAT} \Leftrightarrow f(x) \in \mathrm{SAT}$).

   Show that if P $\neq$ NP, then $|f(x)| \geq x$ for some string $x \in \{0,1\}^*$.

b. Let $H\colon \mathbb{N} \to \mathbb{N}$ be an unbounded monotonically increasing function. Suppose the function $x \mapsto H(|x|)$ is poly-time computable.

   Show that if P $\neq$ NP, then SAT_H is not NP-complete.

[1] https://en.wikipedia.org/wiki/Exponential_time_hypothesis