Each question is worth 20 points.

**Question 1**

Consider the following decision problem:

**MIN-Circuit**: Given an $n$-input 1-output Boolean circuit $C$, decide if $C$ is a circuit of minimum size computing the function $x \mapsto C(x)$.

Show that $\text{MIN-Circuit} \in \text{coNP}^{\text{SAT}}$. (In fact, this problem is $\text{coNP}^{\text{SAT}}$-complete with respect to polynomial-time Karp reductions.)

**Question 2**

We say that an $n$-input 1-output circuit $C$ is a *formula* if every gate in $C$ has out-degree 1.

a. Show that every function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ has a formula of size $O(2^n)$.

b. Show that for every large enough $n \in \mathbb{N}$, there exists a function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ that requires formulas of size $\Omega(2^n / \log n)$.

**Question 3**

Show that every function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ has a circuit of size $O(2^n / n)$. (Also see the hints for exercise 6.1 in the textbook.)